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# Low-temperature thermal properties of yttrium and lutetium dodecaborides

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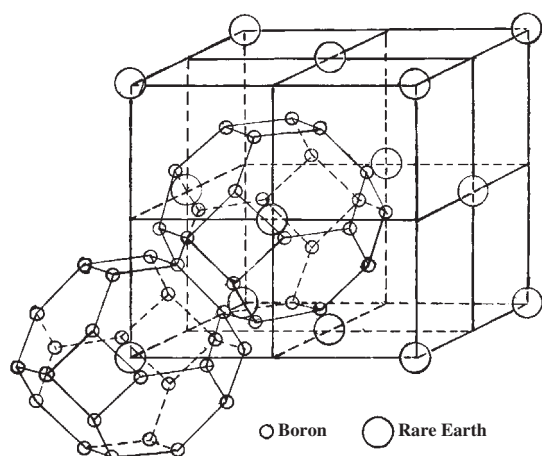
## Abstract

The heat capacity ( $C_p$ ) and dilatation ( $\alpha$ ) of  $\text{YB}_{12}$  and  $\text{LuB}_{12}$  are studied.  $C_p$  of the zone-melted  $\text{YB}_{12}$  tricrystal is measured in the range 2.5–70 K, of the zone-melted  $\text{LuB}_{12}$  single crystal in the range 0.6–70 K, and of the  $\text{LuB}_{12}$  powder sample in the range 4.3–300 K;  $\alpha$  of the zone-melted  $\text{YB}_{12}$  tricrystal and  $\text{LuB}_{12}$  single crystals is measured in the range 5–200 K. At low temperatures a negative thermal expansion (NTE) is revealed for both compounds: for  $\text{YB}_{12}$  at 50–70 K, for  $\text{LuB}_{12}$  at 10–20 K and 60–130 K. Their high-temperature NTE is a consequence of nearly non-interacting freely oscillating metal ions (Einstein oscillators) in cavities of a simple cubic rigid Debye lattice formed by  $\text{B}_{12}$  cage units. The Einstein temperatures are  $\sim 254$  and  $\sim 164$  K, and the Debye temperatures are  $\sim 1040$  K and  $\sim 1190$  K for  $\text{YB}_{12}$  and  $\text{LuB}_{12}$  respectively. The  $\text{LuB}_{12}$  low-temperature NTE is connected with an induced low-energy defect mode. The  $\text{YB}_{12}$  superconducting transition has not been detected up to 2.5 K.

## 1. Introduction

Boron forms refractory dodecaborides ( $\text{MeB}_{12}$ ) with heavy lanthanides from terbium to lutetium, and also with scandium, yttrium, zirconium, uranium. Apart from  $\text{ScB}_{12}$ , they crystallize in the face-centred cubic  $\text{UB}_{12}$ -type structure (space group  $Fm\bar{3}m-O_h^5$ ), which may be considered as a cubic lattice formed by rigid units— $\text{B}_{12}$  cubooctahedra—with interstitial metallic atoms accommodated in the octahedral pores among them or as a structure in which

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**Figure 1.** The  $UB_{12}$ -type crystal structure of metal dodecaborides.

metallic atoms are located at the centres of the regular Fedorov's cubooctahedra ( $MeB_{24}$ ) with boron atoms at each their 24 vertices (figure 1).

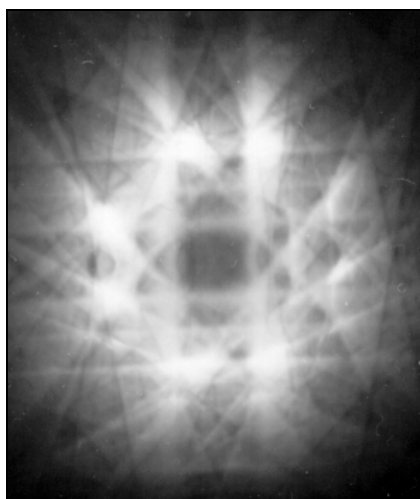
Strong covalent bonds between boron atoms of inter- and intra- $B_{12}$  cages lead to a very rigid  $MeB_{12}$  lattice. The size of the metal atom has a weak effect on the unit cell dimension: when the radius of metallic ions is changed from 0.80 to 0.97 Å ( $\sim 17\%$ ) the unit cell dimension varies from 7.407 to 7.501 Å ( $\sim 1.3\%$ ) [1]. However, the presence of metal atoms stabilizes this structure and in particular the cubooctahedral form of the  $B_{12}$  group due to transferring to the cubooctahedron two valence electrons per metallic atom in order to compensate the electron deficiency in the boron sublattice.

In trivalent metal dodecaborides the third valence electron enters the conduction band that in combination with the rigid  $MeB_{12}$  lattice determines their good metal properties. Diamagnetic dodecaborides— $ScB_{12}$ ,  $YB_{12}$ ,  $ZrB_{12}$  and  $LuB_{12}$ —are superconductors with ordering temperatures of 0.39, 4.7, 5.82 [2] and 0.4 K [3], respectively. Here it is necessary to remark that for  $YB_{12}$  the value of  $T_c = 4.7$  K in this study has not been confirmed.  $YbB_{12}$  belongs to an interesting class of Kondo insulators [4]. The remaining dodecaborides with unfilled 4f-shell are antiferromagnets at low temperatures [5].

At average dimension of the  $B_{12}$  cubooctahedron of about 5.35 Å, the radius of the  $B_{24}$  cavity changes in the range 1.15–1.2 Å [1]. Taking into account the size of the metal ions it may be supposed that the oscillations of the metal ions (Einstein oscillators) are sufficiently free around their sites, which may result in the appearance of a quasi-local optical mode with the energy inside the Debye spectrum and a deformation of the long-wave range of the spectrum that in its turn has to be reflected in phonon-depending properties. Experimental information on the phonon spectrum of dodecaboride exists only for  $YbB_{12}$  [6, 7] and for  $LuB_{12}$  [6–9]. It is assumed that their lowest peaks at  $\sim 15.5$  and  $\sim 14$  meV correspond to the flat mode from non-interacting vibrations of the Me ions while the higher ones are mostly due to the boron optical and rotational modes in the middle energy range and intramolecular B–B vibrations in the highest-energy part.

To shed light on the possible manifestation of such quasi-local optical modes in the crystal properties we have undertaken studies of the thermal properties (heat capacity and thermal expansion) of non-magnetic yttrium and lutetium dodecaborides, which are determined only by the outer electrons and by the phonon spectrum.

The previously published data on the thermal expansion of  $YB_{12}$  and  $LuB_{12}$  in the temperature range from 78 up to 1200 K show a monotonic increase of the thermal expansion



**Figure 2.** The symmetry electron Kikuchi pattern to the [100] direction for the LuB<sub>12</sub> single crystal.

coefficient ( $\alpha$ ) with temperature. These results were obtained using the x-ray method [10, 11] and quartz dilatometers [10, 12] on powders and compact sintered samples. Information on the heat capacity has been reported only for LuB<sub>12</sub> up to 300 K [4]; its heat capacity is also characterized by a monotonic rise.

Special attention has been paid in this work to the quality of the samples studied.

## 2. Samples and experimental methods

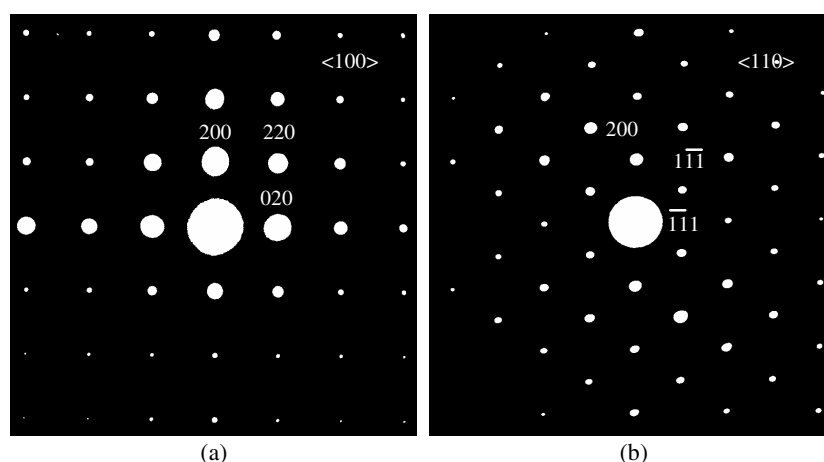
The process of sample preparation consists of the synthesis of dodecaborides by a borothermal reduction of the metal oxides in vacuum at 1900 K, the compacting of these powders into rods and their subsequent sintering, and of inductive zone melting.

The purity of the initial oxides Y<sub>2</sub>O<sub>3</sub> and Lu<sub>2</sub>O<sub>3</sub> was 5N and 4N, respectively. The content of the main substance in the initial amorphous boron was no less than 99.5%. Highly volatile impurities in boron were deleted partially during the synthesis procedure and partially during zone melting. The total content of impurities in samples studied was not higher than 10<sup>-3</sup> mass%.

A part of the LuB<sub>12</sub> synthesized powder was used for its heat capacity study in the range 4.3–300 K. The composition corresponds to the stoichiometric one; the lattice parameter is equal to 7.4644<sub>2</sub> Å.

The thermal expansion 5–200 K and low-temperature heat capacity (up to 70 K) studies were performed on samples which were cut from the zone-melted YB<sub>12</sub> tricrystal and on LuB<sub>12</sub>⟨100⟩ and ⟨110⟩ single crystals. The lattice parameter of the LuB<sub>12</sub> single crystal coincides with the powder one due to the rigid boron sublattice; the YB<sub>12</sub> tricrystal lattice parameter is equal to 7.500<sub>1</sub> Å in accordance with published data [1].

The ⟨100⟩ and ⟨110⟩LuB<sub>12</sub> single crystals were grown using correspondingly oriented seeds. According to the x-ray topography and Laue photograph analysis their real structures are highly perfect. Electron diffraction and high-resolution transmission microscopy analysis allowed us to determine the characteristics of local structural details of the single crystals: according to the electron Kikuchi patterns (figure 2) and point electron diffraction patterns (figures 3(a), (b)), which correspond only to the UB<sub>12</sub>-type structure, defects were practically absent.



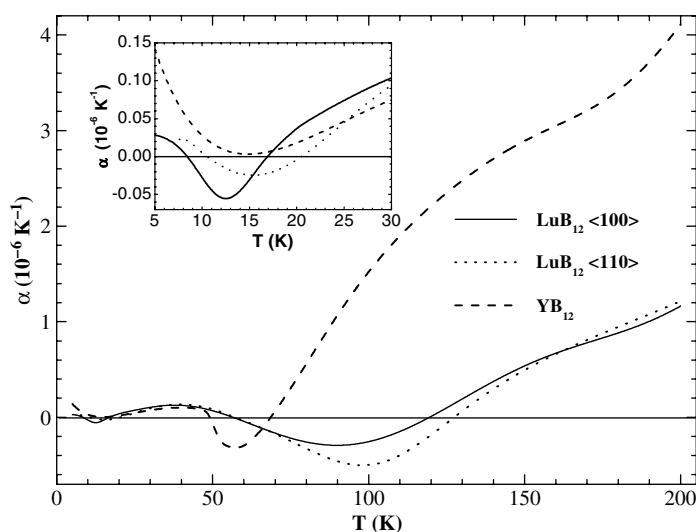
**Figure 3.** Electron diffraction patterns for two  $\text{LuB}_{12}$  single crystals along their main zone axes: (a)  $\langle 100 \rangle$ , (b)  $\langle 110 \rangle$ . All reflections correspond to the  $\text{UB}_{12}$ -type structure.

However, it is impossible to exclude the presence of microdefects in these crystals, as at densities less than  $10^6 \text{ cm}^{-2}$  the probability of their presence in the area of the visible field is low. A possible existence of defects follows also from the chemical analysis data and the hydrostatic density measurements of the samples. According to them, the composition of the lutetium dodecaboride single crystals may be estimated as  $\text{LuB}_{11.89 \pm 0.06}$ . Obviously, this composition is at equilibrium for the crystallization temperature. Any attempt to obtain single crystals with a nominal stoichiometry by the input of surplus boron to the initial powders resulted in an instability of the growing process and as a consequence to the obtaining of mosaic crystals.

The yttrium dodecaboride zone-melted sample represented a tricrystal with approximate composition  $\text{Y}_{0.90}\text{B}_{12}$  in agreement with the results of Johnson and Daane [13]: melted yttrium dodecaboride exists only for  $\text{Y}_x\text{B}_{12}$  compositions where  $x \leq 0.92$ . This compound is located on the border of the  $\text{UB}_{12}$ -type structure stability; therefore the presence of defects in the lattice allows the cubic crystal structure to stabilize.

The heat capacity ( $C_p$ ) was measured by the pulse quasi-adiabatic method for the  $\text{YB}_{12}$  tricrystal in the range 2.5–70 K, for the  $\text{LuB}_{12}$  single crystal in the range 0.6–70 K and for the  $\text{LuB}_{12}$  powder sample from 4.3 to 300 K using respectively two types of calorimeter (for compact samples and for the powder one) which have been described in detail in [14, 15]. The  $\text{YB}_{12}$  heat capacity below 6 K was measured also in a magnetic field of 1 T assuming the possibility of the superconductive transition in  $\text{YB}_{12}$  at 4.7 K [2]. The mass of the  $\text{LuB}_{12}$  powder sample was about 25 g, and the masses of the zone-melted  $\text{YB}_{12}$  and  $\text{LuB}_{12}$  samples were less than 400 mg. The experimental error of the  $C_p$  study by a copper standard for the calorimeter with the compact samples is less than 1% in the  $0.6 \pm 70 \text{ K}$  range [14]; in the case of the calorimeter for the powder sample the experimental error compared to a leucosapphire standard is equal to 1.46% in the range  $4.3 \pm 30 \text{ K}$ , 0.53% at  $30 \pm 100 \text{ K}$ , and 0.31% at  $100 \pm 300 \text{ K}$  [15].

The temperature dependence of the relative length change ( $\Delta l/l$ ) was measured by the three-terminal capacitive method in the range 5–200 K with a resolution  $1 \times 10^{-9}$ . The capacitive cell was very similar to the cell described by Brandli and Griessen [16]. The experimental  $\Delta l/l$  curves were fitted by a polynomial after subtraction of the experimental cell contribution. The linear thermal expansion coefficient  $\alpha = (\partial \ln l / \partial T)_p$  was then



**Figure 4.** The temperature dependence of the thermal expansion coefficient  $\alpha$  for the  $\text{YB}_{12}$  tricrystal and the  $\text{LuB}_{12}\langle 100 \rangle$  and  $\langle 110 \rangle$  single crystals. Inset:  $\alpha$  versus  $T$  in an expanded scale.

determined by differentiating the polynomial curve obtained. The relative experimental error  $\Delta l/l$  compared to standard copper is less than  $\pm 1\%$  below 20 K and decreases down to 0.3% near 200 K. The samples were cylinders or parallelepipeds ( $l = 5\text{--}6$  mm, diameter  $\sim 4$  mm) with plane-parallel basis (precision  $\pm 1\ \mu\text{m}$ ).

### 3. Results and discussion

The temperature dependences of the thermal expansion coefficients  $\alpha$  for the  $\text{YB}_{12}$  tricrystal and the  $\text{LuB}_{12}$  single crystals are shown in figure 4. Both compounds reveal a negative thermal expansion (NTE). In the case of  $\text{LuB}_{12}$  there are two intervals of NTE: the ‘low-temperature’ one with a minimum at about 12 K and the ‘high-temperature’ one with a minimum at about 90 K. The  $\text{YB}_{12}$  thermal expansion reveals only the ‘high-temperature’ NTE with a minimum at about 60 K; instead of the ‘low-temperature’ NTE only an  $\alpha$  minimum occurs at  $\sim 15$  K. The  $\text{YB}_{12}$  ‘high-temperature’ NTE anomaly is shifted to lower temperatures in comparison with the  $\text{LuB}_{12}$  one.

There is a slight difference in the thermal expansion of the  $\text{LuB}_{12}$  single crystals with  $\langle 100 \rangle$  and  $\langle 110 \rangle$  orientations. Because of technical reasons it was impossible to cut two samples along  $\langle 100 \rangle$  and  $\langle 110 \rangle$  axes from the same single crystal ingot for the  $\Delta l/l$  measurements. The possible origins of the difference in the values of  $\alpha$  are (1) high sensitivity of the thermal expansion of  $\text{LuB}_{12}$  to the quality of the single crystals grown, (2) the anisotropy of the chemical bonds (the  $\langle 100 \rangle$  axis corresponds to the Me–B bond direction and the  $\langle 110 \rangle$  axis to the Me–Me bond) and/or (3) the distortion of the  $\text{LuB}_{12}$  crystal structure at low temperatures from the cubic one. An x-ray study of the  $\text{LuB}_{12}$  crystal structure down to low temperatures (about 10 K) is in progress.

For these two compounds the coefficient  $\alpha$  is a sum of the phonon  $\alpha_{\text{ph}}$  and electron  $\alpha_{\text{e}}$  contributions. The sign of  $\alpha$  is determined by the sign of the effective Grüneisen parameter  $\Gamma_{\text{eff}}$

$$\alpha = \Gamma_{\text{eff}} C_v / 3BV \quad (1)$$

because the heat capacity at constant volume  $C_v$ , isothermal bulk modulus  $B$  and molar volume  $V$  are positive quantities.

The measured  $\Gamma_{\text{eff}}$  is the average of the electron  $\Gamma_e$  and phonon  $\Gamma_{\text{ph}}$  Grüneisen parameters weighted with the respective heat capacities—the electron ( $C_e$ ) and the phonon ( $C_{\text{ph}}$ ) ones:

$$\Gamma_{\text{eff}} = (\Gamma_e C_e + \Gamma_{\text{ph}} C_{\text{ph}}) / C_v. \quad (2)$$

Conduction electrons give a contribution to the effective Grüneisen parameter only at low temperatures, when  $C_e$  becomes comparable with  $C_{\text{ph}}$ . The  $\text{YB}_{12}$  and  $\text{LuB}_{12}$  high-temperature NTEs are located in the middle temperature range. So we have supposed that this anomaly is determined by dodecaboride phonon spectra peculiarities.

The phonon Grüneisen parameter  $\Gamma_{\text{ph}}$  is weighted average of all  $\Gamma_i$ :

$$\Gamma_{\text{ph}} = \frac{\sum_i \Gamma_i C_i}{\sum_i C_i} \quad (3)$$

where

$$\Gamma_i = -\frac{d \ln \omega_i}{d \ln V} \quad (4)$$

is the Grüneisen parameter of the  $i$ th frequency mode, the sign of which is determined by the sign of the  $\partial \omega_i / \partial V$  derivative and may be both positive and negative;  $C_i$  is the heat capacity of the  $i$ th mode. As the total  $\Gamma_{\text{ph}}$  is determined by contributions from all phonon modes, the sign of  $\Gamma_{\text{ph}}$  may be positive or negative depending on correlation among individual positive and negative  $\Gamma_i$ -value contributions [17].

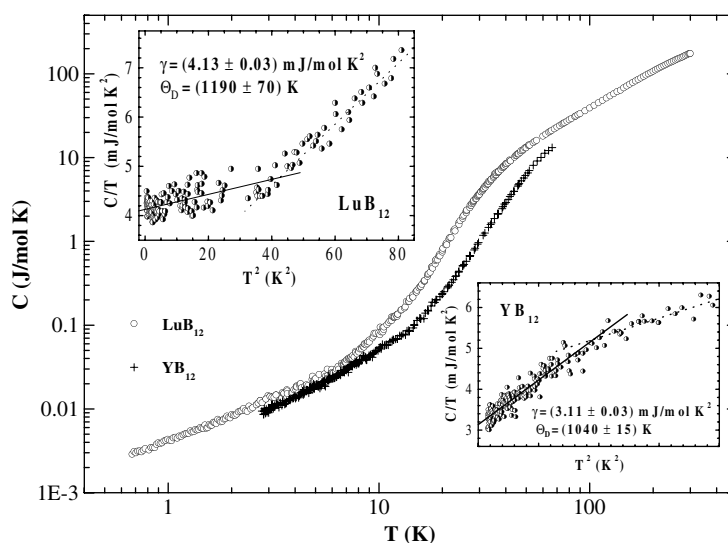
As we show below, one may obtain the necessary information on low-lying states of the phonon spectra of  $\text{YB}_{12}$  and  $\text{LuB}_{12}$  from the temperature dependence of their lattice heat capacity.

The temperature dependence of the heat capacities of both dodecaborides is shown in figure 5. Up to 2.5 K we did not find any evidence of the  $\text{YB}_{12}$  superconductive transition: the heat capacity in the magnetic field and without it coincided. No transition is seen on the thermogram. It is possible that the superconductive transition detected by Mathias *et al*  $\text{YB}_{12}$  was induced by  $\text{YB}_6$  impurities with  $T_c = 7.1$  K [2]. There is an additional feature on the  $\text{YB}_{12}$  heat capacity curve: at low temperatures it is an extremely small value and as a consequence there is data scattering. In order to be sure that the  $\text{YB}_{12}$  low-temperature heat capacity, which we analyse in low-temperature model, corresponds to the normal state and not the superconducting one, we have treated the heat capacity data obtained in magnetic field ( $H = 1$  T) at  $T < 6$  K (figure 5). The heat capacity of  $\text{YB}_{12}$  and  $\text{LuB}_{12}$  is a sum of two contributions—the electron ( $C_e$ ) and phonon ( $C_{\text{ph}}$ ) ones. At low temperatures

$$C_p = \gamma T + bT^3, \quad (5)$$

where  $\gamma$  and  $b$  are coefficients of the renormalized electronic heat capacity and of phonon heat capacity, respectively.  $b$  is equal to  $\{(12/5)rR\pi^4/\Theta_D^3\}$  with  $r = 13$  being the number of atoms in the molecule of the  $\text{UB}_{12}$  structure type. From the plot of  $C_p/T$  versus  $T^2$  (insets in figure 5) we determined the Sommerfeld coefficient  $\gamma$  and the Debye temperature  $\Theta_D$ . They are equal to  $4.13 \pm 0.03$  mJ mol<sup>-1</sup> K<sup>-2</sup> and  $1190 \pm 70$  K for  $\text{LuB}_{12}$  and  $3.11 \pm 0.03$  mJ mol<sup>-1</sup> K<sup>-2</sup> and  $1040 \pm 15$  K for  $\text{YB}_{12}$ .

The estimated  $\gamma$ -value for  $\text{LuB}_{12}$  is close to  $\gamma \approx 4.2$  mJ mol<sup>-1</sup> K<sup>-2</sup> reported by Iga *et al* [18]. The non-renormalized  $\text{LuB}_{12}$   $\gamma_0$ -value is equal to  $2.85$  mJ mol<sup>-1</sup> K<sup>-2</sup> [19]. The difference between  $\gamma$  and  $\gamma_0$  is caused by the enhancement of the effective mass of conduction electrons due to their coupling with phonons [20]. The experimental enhancement factor (the electron–phonon coupling)  $\lambda_{\text{ph-e}} = (\gamma/\gamma_0) - 1 = 0.45$ . We estimated  $\lambda_{\text{ph-e}}$  using equations



**Figure 5.** The temperature dependence of the  $\text{YB}_{12}$  and  $\text{LuB}_{12}$  heat capacity. Insets:  $C/T$  versus  $T^2$  for  $\text{YB}_{12}$  and  $\text{LuB}_{12}$ .

obtained for two cases: (a) for the weak-coupling limit and Einstein phonon spectra [21] and (b) for the extended spectrum with a Debye temperature [22]. For the superconducting transition temperature  $T_c = 0.4$  K [3] respectively the calculated  $\lambda_{\text{ph-e}}$ -values are equal to  $0.39 \pm 0.05$  and  $0.35 \pm 0.05$  for  $\mu^* = 0.13 \pm 0.03$  for two characteristic temperatures  $\Theta_E = 164$  K (see below) and  $\Theta_D = 1190$  K.

For  $\text{YB}_{12}$ ,  $\gamma_0$  is equal to  $2.36 \text{ mJ mol}^{-1} \text{ K}^{-2}$  [19],  $\lambda_{\text{ph-e}} = 0.32$ . As was remarked above, the  $\text{YB}_6$  superconducting transition has not been found up to 2.5 K. The estimated  $\lambda_{\text{ph-e}}$  are equal to 0.46 or 0.57 for the upper experimental limit  $T = 2.5$  K and  $\Theta_D = 1040$  K or  $\Theta_E = 254$  K respectively and are far from experimental  $\lambda_{\text{ph-e}} = 0.32$ , which is additional evidence that the above-mentioned value  $T_c = 4.7$  K is unreal.

The estimated  $\Theta_D$  for  $\text{YB}_{12}$  and  $\text{LuB}_{12}$  are close to  $\Theta_D$  for  $\beta$ -boron determined by the x-ray method (1250 K) [12] and calculated from the elastic constants (1370 K) [23], which suggests that the strengths of the bonding in these solids are similar. Unfortunately, a large scattering of experimental points in the low-temperature range due to the very small heat capacity of  $\text{LuB}_{12}$  did not allow us to determine its  $\Theta_D$ -value more accurately.

For both compounds  $C_p/T$  versus  $T^2$  plots reveals a knee at low temperatures (insets in figure 5). Such behaviour of  $C_p$  was found also for rare earth hexaborides [24]. Takegahara and Kasuya [25] supposed that such an anomaly in the hexaboride case is due to the defect-induced soft mode. Calculations of the  $\text{LaB}_6$  heat capacity from its point contact (PC) spectra, which included a zero bias anomaly usually connected with impurities and defects, gave a similar knee in the  $C_p/T$  versus  $T^2$  plot [26] confirming Takegahara and Kasuya's idea.

Now we try to extract information on the phonon spectra of yttrium and lutetium dodecaborides from their heat capacities.

According to Junod *et al* [27],  $5/4R\pi^4 C_{\text{ph}}/T^3$  versus  $(T/1 \text{ K})$  gives an approximate picture of the one-dimensional phonon density of states  $\omega^{-2}F(\omega)$  for  $\omega = 4.928T$ , where  $\omega$  is expressed in kelvins. On a logarithmic scale the response of  $C_{\text{ph}}/T^3$  to a  $\delta$  function (Einstein peak) is a bell-shaped peak.



**Table 1.** The frequencies of the LuB<sub>12</sub> Raman modes (meV).

Modes					
F <sub>2g</sub>	E <sub>g</sub>	F <sub>2g</sub>	E <sub>g</sub>	A <sub>g</sub>	Ref.
56.34	83.42	102.6	125.8	134.6	[7]
	82.20	97.20		129.3	[9]

**Table 2.** The Einstein frequencies ( $\Theta_{E_i}$ ) for LuB<sub>12</sub>.

Nos	Heat capacity (this work)			Neutron [6]		PC spectrum [8]	
	K	meV	Degeneracy	K	meV	K	meV
1	58.0	4.9	0.049			(31.3) <sup>a</sup>	(2.7)
2	163.7	14.1	3	164.8	14.2	168.3 (163.6)	14.5 (14.1)
3	307.5	26.5	2	273.2	23.6	280.8	24.4
4	374.4	32.3	1	374.6	32.3	338.9	29.2
5	513.0	44.2	3	491.3	42.4	436.3	37.6
6	743.0	64.0	3	579.5	49.9		
7	843.0	72.64	3				
8	990.0	85.31	3				
9	1405.0	121.0	7				

<sup>a</sup> The energies in brackets from the PC spectra are obtained for high-resistance contacts; the other ones are for low-resistance contacts [8].

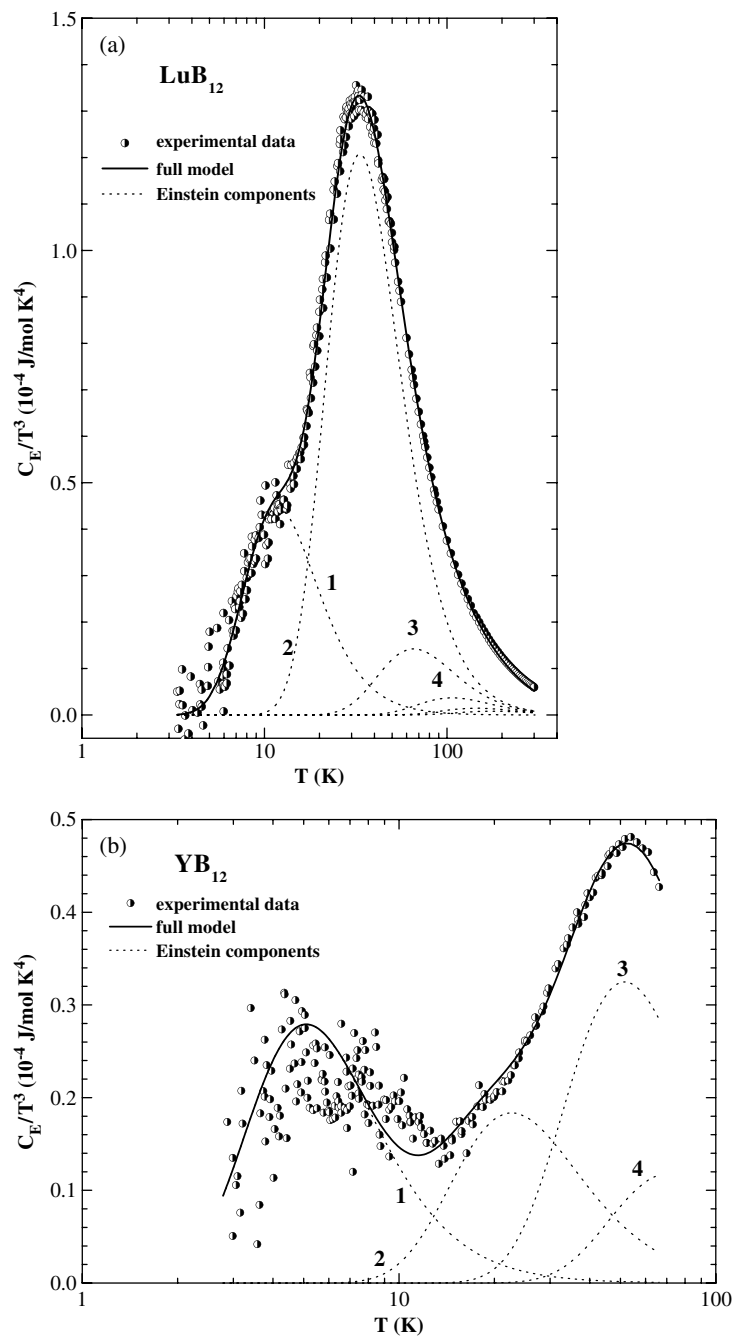
After elimination of the electron heat capacity ( $C_e = \gamma T$ ,  $\gamma = \text{const}$ ) such an analysis of the phonon heat capacity ( $C_{\text{ph}} = C_p - C_{\text{el}}$ ) was carried out for both compounds. We have found well-defined bell-shaped peaks, with maxima corresponding to the phonon modes with characteristic energies of ( $\omega_E \approx 5T_m$ ) about 14.0 and 22 meV for LuB<sub>12</sub> and YB<sub>12</sub>, respectively (figures 6(a), (b)—circles). The obtained energy of about 14 meV for LuB<sub>12</sub> agrees with the flat mode from free vibrations of the Lu ion obtained from the neutron and PC spectra [6, 8] (table 2).

By analogy with results for LaB<sub>6</sub> [28], we consider the molecule of UB<sub>12</sub> structure type as a diatomic molecule, where one atom is the metal and other is the B<sub>12</sub> structure unit, and calculated the heat capacity of LuB<sub>12</sub> as a superposition of electron ( $C_{\text{el}}$ ), Debye ( $C_{\text{D}}$ ) and single Einstein ( $C_{\text{E}}$ ) contributions. We linked the Debye heat capacity with the B sublattice and calculated it for 12 moles of the B ions; the Einstein heat capacity was attributed to the metal sublattice and respectively to 1 mol of the Lu ions. For these calculations we used the specific  $\Theta_{\text{D}}$ -value from the  $C_p/T$  versus  $T^2$  plot.

The value of  $\Theta_{\text{E}}$  was found from the peak value of the  $C_{\text{ph}}/T^3$  versus  $\ln(T/K)$  curve.

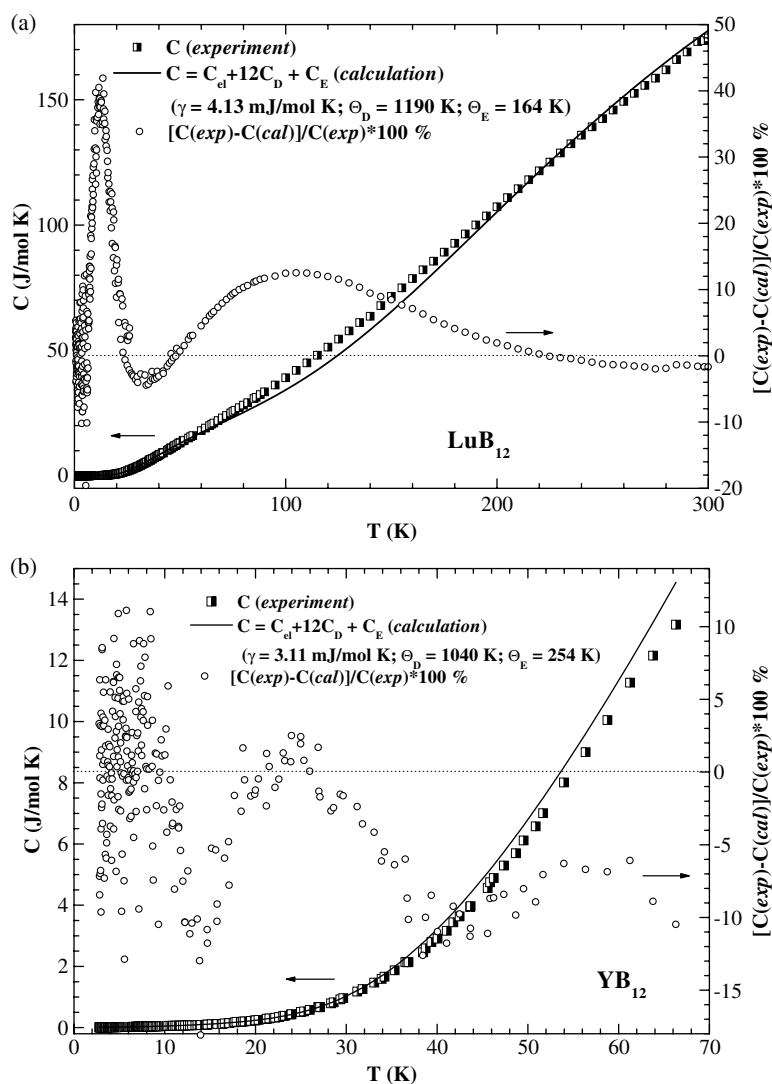
The experimental and calculated LuB<sub>12</sub> heat capacities show satisfactory agreement; however, the existence of a structure in the remainders  $\{[C_p(\text{exp}) - C_p(\text{cal})]/C_p(\text{exp}) \cdot 100\%$  (figure 7(a)) testifies that the agreement is only apparent, and correspondingly the given model is inadequate to explain the experimental data. The strongest disagreement is observed for the low-energy part. a similar  $C_p$  behaviour is observed for YB<sub>12</sub> (figure 7(b)).

The solution of the inverse task—the reconstruction of the phonon spectrum from the heat capacity—is a very difficult problem. For a molecule having 13 atoms, such as the dodecaboride one, in the case of completely lifted degeneracy the number of modes has to be equal to 39; three acoustic modes are related with the Debye contribution to the heat capacity, and the remaining optic ones are Einstein contributions. For LuB<sub>12</sub> the values of its five high-energy modes (2F<sub>g</sub>, 2E<sub>g</sub>, A<sub>g</sub>) are known from its Raman spectra (table 1) [7, 9]; accounting for their degeneracy we still have 11 modes.



**Figure 6.**  $C_{\text{ph}}/T^3$  versus  $\log T$  for  $\text{LuB}_{12}$  (a) and  $\text{YB}_{12}$  (b). The circles are experimental data, and the lines are fitted curves. For  $\text{LuB}_{12}$  the symbols (1–4) correspond to the next energies of the Einstein modes, 58.0, 163.7, 307.5, 374.4 K; for  $\text{YB}_{12}$  the symbols (1–4) correspond to energies 25.5, 111.4, 254, 359.7 K.

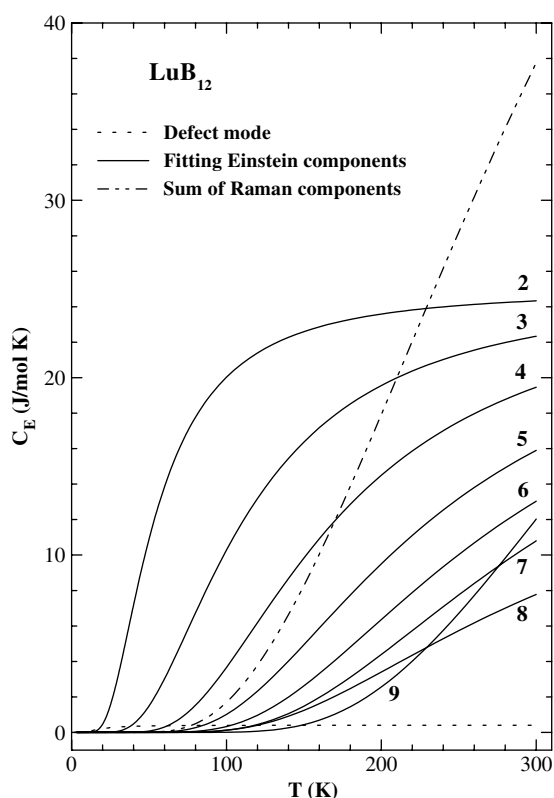
In order to evaluate the remaining 25 modes, some of which may be degenerated, we subtracted Raman and Debye contributions from the  $\text{LuB}_{12}$  full phonon heat capacity and



**Figure 7.** The temperature dependence of the experimental and calculated (using the model of a bi-atomic molecule; see the text)  $\text{MeB}_{12}$  heat capacity and of the relative error between them: (a)  $\text{LuB}_{12}$ , (b)  $\text{YB}_{12}$ .

fitted the remaining heat capacity ( $C_E$ ) by a set of Einstein components ( $\Theta_{E_i}$ ). The results of this fitting are presented in figure 6(a) and table 2; in the latter the data from neutron experiment and PC spectra are also presented. Taking into account that on this scale only the contribution of low-lying modes is clearly seen, in figure 8 all components are presented on the usual scale  $C = f(T)$  in order to evaluate their contribution to the heat capacity in different temperature ranges.

The agreement between experimental and fitted  $\Theta_{E_i}$ -values (table 2) for the low-energy modes is satisfactory; it is necessary to consider the values of high-energy modes as estimates. Any divergence of the modes from the PC spectra may be attributed to the anisotropy of the  $\text{LuB}_{12}$  phonon spectrum which is known to exist for the other boride,  $\text{LaB}_6$  [26]. Unfortunately



**Figure 8.** The temperature dependences of contributions of the individual Einstein phonon modes to the  $\text{LuB}_{12}$  heat capacity. The numbers at the curves correspond to the numbers of the Einstein components in table 2.

information on the  $\text{LuB}_{12}$  PC spectrum is presented only for a unique orientation of the sample [8]. Spectra obtained from data on the neutron scattering and heat capacity are averaged ones, in contrast to the PC spectra.

Let us examine the information from figure 6(a).

In order to obtain the best fitting in the low-energy part of the  $\text{LuB}_{12}$  heat capacity we introduced an additional Einstein contribution with a small weight ( $\sim 0.05$ ) and energy  $\sim 4.9$  meV (figure 6(a), table 2). This contribution to  $C_{\text{ph}}$  is very small in general but very essential at low temperatures. The calculations of the electron heat capacity for the Grimvall model [29] leads to similar results<sup>6</sup>. This low-energy mode corresponds to the zero-bias anomaly in the PC spectrum (table 2). So, we assume that it is a defect-induced soft mode, and that the ‘low-temperature’ NTE and the anomalous low-temperature dependence of the  $\text{LuB}_{12}$  heat capacity have the same origin—a possible formation of the two-level tunnelling system based on the metal ions and defects [25]. Earlier, an identical mechanism has been proposed for the explanation of the low-temperature NTE in  $\text{LaB}_6$  single crystals [30].

<sup>6</sup> Grimvall presented the calculated temperature dependence of the electron–phonon renormalization contribution  $\gamma_{\text{ep}}(T)/\gamma_{\text{ep}}(0)$  to the electronic heat capacity as a plot. The calculation was done for an Einstein phonon spectrum. The temperature dependence is non-monotonic; its distinguishing feature is the peak  $\gamma_{\text{ep}}(T)/\gamma_{\text{ep}}(0)$  at a low temperature, such as  $T/\theta_{\text{E}} \sim 0.1$ . Taking into account the relatively small energy of the first Einstein mode we wanted to exclude the possible effect of the electron heat capacity’s nonlinear character.

The main contribution to  $C_{\text{ph}}$  for  $\text{LuB}_{12}$  at low temperatures is related to the mode equal to 14.1 meV (figure 6(a)) that is very close to the energy of the  $\delta$ -like peak from the neutron phonon spectrum (14.2 meV) [6] and from the PC spectrum (14.5 meV) [8]. The use of this phonon mode resulted in the estimate  $\lambda_{\text{ph-e}} = 0.39$  from the equation for  $T_c$  in the weak-coupling limit and for Einstein phonon spectra [21]. This value is close to the experimental value  $\lambda_{\text{ph-e}} = 0.45$ ; however, to say unambiguously that the electron–phonon coupling constant  $\lambda_{\text{ph-e}}$  is determined first of all by the interaction of electrons with the low-energy first mode in the phonon spectrum is impossible, as  $\lambda_{\text{ph-e}} = 0.35$  from the formula for extended spectrum with a Debye temperature [22] has a similar value.

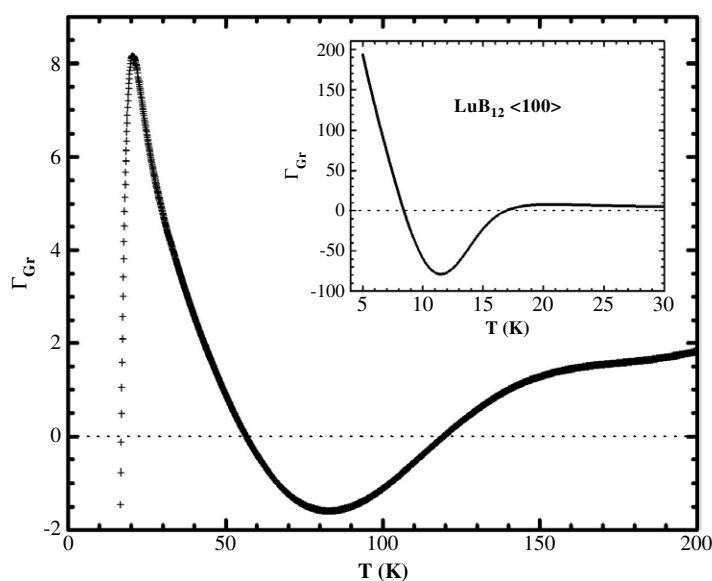
The  $\text{LuB}_{12}$  phonon spectrum from [6, 8] is very similar to that for  $\text{LaB}_6$  [25, 31], in which the first  $\delta$ -like peak at 12.4 meV corresponds to the flat acoustic longitudinal and transverse modes in the larger part of the first Brillouin zone. It is supposed that the flattening of  $\text{LaB}_6$  dispersion curves is connected with vibrations of the non-interacting La ions. The distance between the metal atoms in  $\text{LuB}_{12}$  is about 20% larger than in  $\text{LaB}_6$  and so the Me–Me bond is weaker in it. Therefore one may assume that acoustic phonon branches for  $\text{LuB}_{12}$  have a similar form as the  $\text{LaB}_6$  ones.

We extended this assumption to the  $\text{YB}_{12}$  phonon spectrum. For  $\text{YB}_{12}$  we had the heat capacity data only up to 70 K. The absence of information on the  $\text{YB}_{12}$  high-energy Raman active modes has been no barrier for deconvolution of its phonon heat capacity to the phonon spectrum as these modes are related to the intramolecular B–B vibrations and have to be similar for both compounds. The evaluation of their contribution to the  $\text{YB}_{12}$   $C_{\text{ph}}$  in this temperature range has shown that it is negligible. Results of deconvolution are presented in figure 6(b). By analogy with  $\text{LuB}_{12}$  we related the mode at 21.9 meV (254 K) with free vibrations of yttrium atoms in the boron cavities. The value of the higher-energy mode is equal to  $\sim 31$  meV with degeneracy 3. However, for more correct fitting of the low-energy part of the spectrum we had to include two additional modes—2.2 and  $\sim 9.6$  meV with weights 0.0025 and 0.14, respectively. We suppose that they have the same nature as the mode with energy  $\sim 4.9$  meV for  $\text{LuB}_{12}$ .

One can explain the softening of the  $\text{LuB}_{12}$  phonon at 14.1 meV in comparison with the  $\text{YB}_{12}$  one at 21.9 meV in the following manner. The 17% difference in the Y and Lu ion radius (0.97 and 0.80 Å relatively) corresponds to the  $\sim 0.5\%$  difference in the  $\text{YB}_{12}$  and  $\text{LuB}_{12}$  lattice parameters (7.500<sub>1</sub> and 7.4644<sub>2</sub> Å, respectively). So the decreasing of the radius of the metal ion located in the centre of the Fedorov cubooctahedron should lead to weakening of the Lu–B bond, certain destabilization of the lattice, softening of the Lu-phonon mode and larger  $\text{LuB}_{12}$  heat capacity than that of  $\text{YB}_{12}$ .

In a first approximation the electron–phonon coupling  $\lambda_{\text{ph-e}}$  is proportional to the vibration amplitude of the metal  $(\hbar/2M_{\text{Me}}\omega_{\text{Me}})^{1/2}$ , where  $M_{\text{Me}}$  and  $\omega_{\text{Me}}$  are the mass and phonon frequency of the metal. The calculation of the Y and Lu metal vibrational amplitudes in both compounds gives close values for their experimental  $M_{\text{Me}}$  and  $\omega_{\text{Me}}$ , which agrees with the fact that both  $\lambda_{\text{ph-e}}$ -values are close to that estimated from McMillan’s formula (0.40 and 0.43 relatively) for  $\Theta_{\text{Ei}}$  equal to 21.9 meV ( $\text{YB}_{12}$ ) and 14.1 meV ( $\text{LuB}_{12}$ ) and the same  $T_c = 0.4$  K. This fact also gives evidence that the  $T_c = 4.7$  K for  $\text{YB}_{12}$  obtained by Matthias *et al* [2] is in error.

The existence of the negative  $\Gamma_i$  for the dispersion-free transverse acoustic phonon modes was predicted for the first time by Dayal [32] and Barron [33]. The prevailing negative contribution of their  $\Gamma_i$  to the general phonon Grüneisen parameter results in the negativity of  $\Gamma_{\text{ph}}$  in definite temperature ranges even for such simple substances as tetrahedral semiconductors Si and Ge [32–35 and references cited therein]. Taking into account the possible dispersionless character of the  $\text{YB}_{12}$  and  $\text{LuB}_{12}$  low-energy phonon modes we suppose



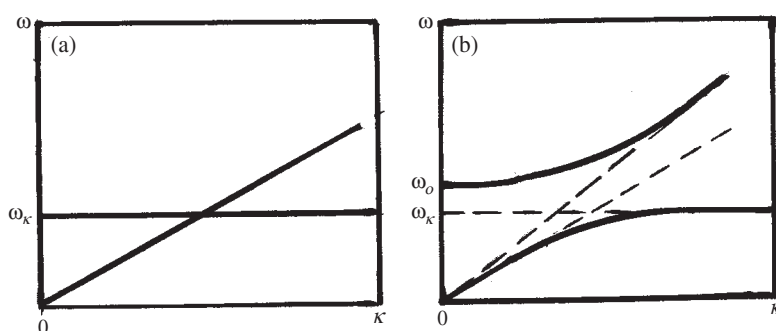
**Figure 9.** The  $\text{LuB}_{12}$  generalized Grüneisen parameter  $\Gamma_{\text{Gr}}$ . Inset:  $\Gamma_{\text{Gr}}$  versus  $T$  for  $\text{LuB}_{12}$  in the expanded low-temperature scale 5–30 K.

that their ‘high-temperature’ NTEs are caused by the negative partial contribution  $\Gamma_i$  from the transverse acoustic modes to a generalized Grüneisen parameter  $\Gamma_{\text{Gr}}$ . For temperatures at which excitations of the transverse phonons with short wavelengths and negative partial contributions  $\Gamma_i$  will dominate, one would expect the NTE.

Due to the alternating signs of the partial Grüneisen parameters and peculiarities in the phonon spectra at 4.2–300 K, the Si and Ge lattice parameters and consequently the thermal expansion coefficients weakly depend on temperature [34]. We think that the small  $\alpha$ -values of dodecaborides are explained by similar reasons in addition to the role of the rigid boron sublattice.

On the ground of the experimental heat capacity, thermal expansion and bulk modulus ( $c_{\text{B}}$ ) data we evaluated the temperature dependence of the  $\text{LuB}_{12}$  generalized Grüneisen parameter  $\Gamma_{\text{Gr}}$ . In figure 9,  $\Gamma_{\text{Gr}}$  is presented for  $\text{LuB}_{12}\langle 100 \rangle$ . In the 10–20 and 60–120 K ranges  $\Gamma_{\text{Gr}}$  for  $\text{LuB}_{12}$  is negative in accordance with its two ranges of NTE, whereas the  $\text{LuB}_{12}$   $C_p$  from 5 K up to 200 K is a monotonically increasing function of temperature (figure 5) and  $c_{\text{B}}$  is a smooth monotonically decreasing function of temperature from 229.5 up to 224.9 GPa, respectively. As a result, the generalized Grüneisen parameter  $\Gamma_{\text{Gr}}$ , but not the heat capacity, mainly determines the  $\text{LuB}_{12}$   $\alpha(T)$  behaviour. As  $c_{\text{B}}$  and  $C_p$  for both  $\text{LuB}_{12}$  samples are the same, the  $\Gamma_{\text{Gr}}$  general structure (two negative minima) is the same for both orientations according to the  $\alpha$  behaviour; the difference between two  $\Gamma_{\text{Gr}}$  is only in details—in the positions of minima and in the  $\Gamma_{\text{Gr}}$  absolute values at the same temperatures. For  $\text{YB}_{12}$  the Grüneisen parameter was not calculated, due to the absence of the bulk modulus data.

We make a last remark. Let us consider a possible mechanism responsible for flattening of the low-lying acoustic branches. Based on the experimental facts, we supposed that the metal ions freely vibrate as harmonic oscillators in the cavities of the boride sublattice. That is, the metal ions behave as a concentrated impurity with a quasi-local frequency. Kossevíc [36] considered the behaviour of such a system and concluded that even a small interaction between vibrations of two subsystems results in two branches of the spectrum



**Figure 10.** A schematic diagram of the phonon dispersion law for a crystal with large impurity concentration: (a) the intersection of the sound dispersion law with the quasi-local frequency of homogeneously distributed impurities; (b) two branches of long-wave vibrations divided by a quasi-gap [36].

of the long-wave vibrations. In figure 10 the scheme of the phonon dispersion law for a crystal with a large impurity concentration is presented: it shows the intersection of the sound dispersion law ( $\omega = s_0k$ ) with the quasi-local frequency of a homogeneously distributed impurity ( $\omega = \omega_k = \text{const}$ ) (figure 10(a)) and formation of two phonon branches with a quasi-gap due to removing of a hypothetical degeneration of frequencies (figure 10(b)) [36]. Under such an approach the formation of flat pieces of dispersion curves in the largest part of the first Brillouin zone becomes clear, and one may comment on deformation of the long-wave range of the phonon spectrum.

#### 4. Conclusion

We studied the thermal expansion and heat capacity of non-magnetic yttrium and lutetium dodecaborides in the low and middle temperature intervals and found the ranges of the negative thermal expansion for both compounds at definite temperatures. The ‘low-energy’ NTE is connected with the possible formation of a two-level tunnelling system based on the metal ions and lattice defects. The NTE at middle temperatures is explained by the features of the phonon structure of these compounds: the flat transverse acoustic phonon modes in the superior part of the first Brillouin zone that results in the negative partial contribution of the Grüneisen parameters  $\Gamma_i$  of these modes to the generalized Grüneisen parameter  $\Gamma_{Gr}$ . The existence of such modes follows from the heat capacity analysis. They appear as a result of oscillations of the non-interacting metal ions in the cavities of the boron sublattice.

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